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VOLTAGE–CURRENT CHARACTERISTIC OF GAS DISCHARGE WITH EXTERNAL IONIZATION

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Steady-state solutions of the discharge equations were obtained in [1] for a range of values of the external ionization source strength Q, discharge current j, and voltage U such that there was no impact ionization in the positive column, and the voltage was below the breakdown value.

The present paper treats a broader range of variation of Q, j, and U; in particular, currents are considered for which impact ionization in the positive column is important, and the voltage is above the breakdown value.

Gas discharge with external ionization can be described by the following equations [1, 2]:

$$\frac{\partial q_-}{\partial t} + \frac{\partial j_-}{\partial x} = \alpha j_- + Q - \beta q_- q_+, \quad \frac{\partial q_+}{\partial t} - \frac{\partial j_+}{\partial x} = \alpha j_- + Q - \beta q_- q_+,$$
$$\frac{\partial E}{\partial x} = \frac{1}{\varepsilon}(q_- - q_+), \quad j_-(0) = \gamma j_+(0), \ j_+(d) = 0, \quad \int_0^d E\,dx = U, \tag{1}$$

where q_- and q_+ are the electron and ion charge densities, j_- and j_+ are the electron and ion current densities, α is the impact ionization coefficient, β is the recombination coefficient, and γ is the coefficient of secondary emission at the cathode resulting from ion impact.

We use the same parameters for nitrogen as in [1]. The pressure is assumed atmospheric, and $\gamma = 0.01$. The method of finding numerical steady-state and transient solutions of system (1) is described in [1].

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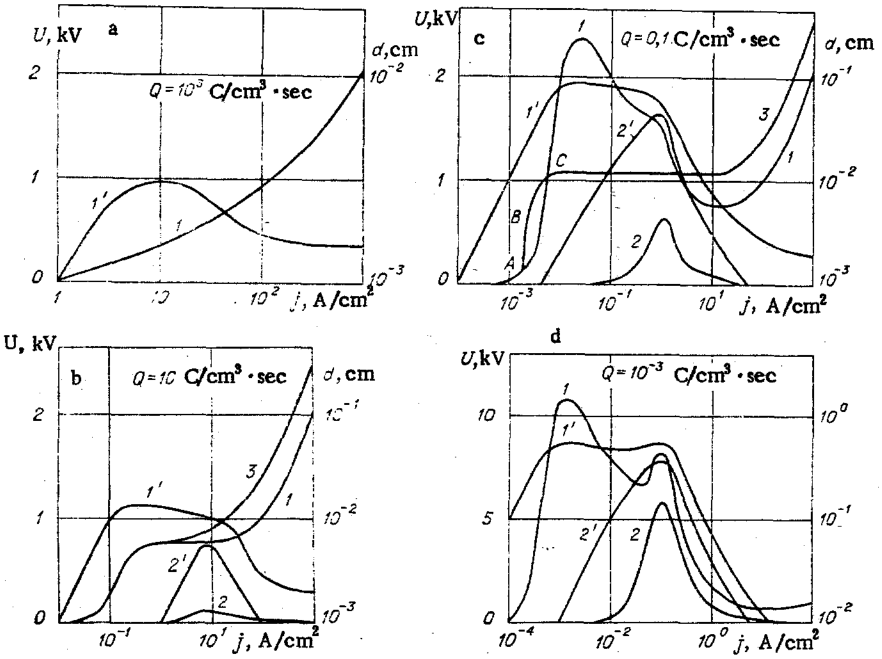

Fig. 1

Figures 1a–d show the cathode fall U_c (curve 1), the anode fall U_a (curve 2), and the cathode and anode arcing distances d_c (curve 1') and d_a (curve 2') with external ionization for an external ionization source strength $Q = 10^3$, 10, 10^{-1}, and 10^{-3} C/cm$^3\cdot$sec as functions of the discharge current j (for $Q = 10^3$ C/cm$^3\cdot$sec the anode fall is not shown since it is negligibly small).

A comparison of Fig. 1a–d shows that for the same current the cathode fall is always larger for smaller values of Q. For $Q \geq 10$ C/cm$^3\cdot$sec the cathode fall increases monotonically with increasing discharge current. Since for $Q \leq 10^{-1}$ C/cm$^3\cdot$sec the cathode fall is a nonmonotonic function of j, for these values of Q the voltage–current characteristics of the discharge can have a section of negative conductivity [1, 2]. For larger currents when $\alpha j_- \gg Q$ in the positive column, the cathode fall does not depend on Q, and increases with increasing current.

A comparison of Fig. 1b–d shows that for the same discharge current the anode fall is always smaller than the cathode fall, and is significant only for relatively small values of Q over a narrow range of j values.

Knowledge of the cathode and anode falls permits the construction of voltage–current characteristics of the discharge for interelectrode distances such that $d_a + d_c < d$. In this case the total potential drop is $U = U_c + U_a + E_0(d - d_c - d_a)$, where E_0 is the electric field intensity in the positive column, and is found from the equation $\alpha j_- + Q - \beta q_- q_+ = 0$.

A comparison of Fig. 1a–d shows that for the same discharge current the sum of d_c and d_a is smaller the larger Q.

We fix the interelectrode distance at d = 0.02 cm.

It is clear from Fig. 1a and b that for $Q \geq 10$ C/cm$^3\cdot$sec, $d_c + d_a < 0.02$ cm over the whole range of j values. Curve 3 of Fig. 1b shows the voltage–current characteristic obtained by the method indicated above.

Figure 1c and d show that for $Q \leq 10^{-1}$ C/cm$^3\cdot$sec the sum of d_c and d_a is greater than 0.02 cm over a certain range of j values. We show how to construct the voltage–current characteristic in this case.

Impact ionization is insignificant for small voltages. Charges are extracted from the cathode sheath and the cathode fall is produced, across which practically the whole applied potential difference occurs. Recombinations can be neglected in the cathode sheath. Then the steady-state solution of (1) which is valid up to the voltage $U = (d^2/2)\sqrt{Qp/\varepsilon\mu_+}$ has the form (the portion up to point A on curve 3 of Fig. 1c):

$$j = (1 + \gamma)\, Q d_c, \quad U = \frac{d_c^2}{2}\sqrt{Qp/\varepsilon\mu_+},$$

where μ_+ is the ion mobility and p is the pressure of the gas. At larger voltages the current density approaches saturation (the portion between points A and B on curve 3 of Fig. 1c):

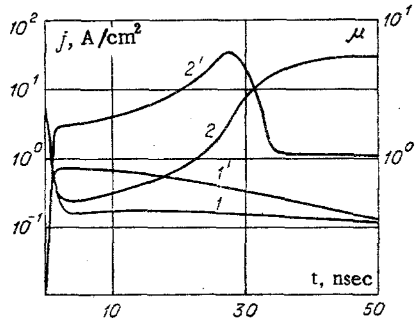

Fig. 2

$$j = (1 + \gamma)Qd.$$

For a further increase in U impact ionization begins to play a role. Since $\partial E/\partial x \ll E/d$, $E = U/d$, it is possible to set α = const. In this case the solution has the form (the portion between points B and C on curve 3 of Fig. 1c)

$$j = \frac{Q}{\alpha} \frac{(1+\gamma)[\exp(\alpha d) - 1]}{1 - \gamma[\exp(\alpha d) - 1]}. \tag{2}$$

The sharp jump in the voltage–current characteristic at the voltage U_* can be understood by considering the temporal evolution of the discharge.

Figure 2 shows the time dependence of the current for $U = 1.05$ kV $< U_*$ (curve 1) and 1.1 kv $> U_*$ (curve 2). At $t = 0$ the charge density $q_- = q_+ = \sqrt{Q/\beta}$ was given. After a time of the order of $\tau = d/v_-$, where $v_- = \mu_- U/pd$, electrons are extracted from the interelectrode gap, the field is distorted, and impact ionization increases. When $U = 1.05$ kV $< U_*$ impact ionization does not completely compensate the process of extraction of ions from the interelectrode gap, the field distortion decreases with time, and the steady-state solution (2) holds. When $U = 1.1$ kV $> U_*$, charges accumulate in the interelectrode gap as a result of impact ionization, the currents increase, and the solution goes over into the quasistationary regime with a positive column and cathode and anode falls.

Figure 2 shows the time dependence of the multiplication factor $\mu = \gamma[\exp(\int_0^d \alpha dx) - 1]$. When $U = 1.05$ kV $< U_*$ (curve 1') μ is always less than 1. When $U = 1.1$ kv $> U_*$ (curve 2') μ becomes greater than 1 during the extraction of electrons.

Thus, the determination of the breakdown voltage U_* is reduced to the solution of the equation $\gamma[\exp(\int_0^d \alpha dx) - 1] = 1$, where the field E is linear in x, and its slope is determined from the initial ion density $q_+ = \sqrt{Q/\beta}$. As $Q \to 0$ the breakdown voltage U_* approaches the breakdown voltage for a uniform field E, which for our parameters is

$$U_{\text{br}} = 1.35 \text{ kV}.$$

For voltages larger than U_* the steady-state solution of (1) enters the branch corresponding to discharge currents for which $\alpha j_- \gg Q$ in the positive column.

We note that the steady-state solution of (1) depends on the initial conditions. This is related to the fact that the breakdown voltage U_* generally depends on the initial charge density and not on Q. If, for example, for $Q = 0.1$ C/cm$^3\cdot$sec the initial charge density is larger than $q_- = q_+ = \sqrt{Q/\beta}$, and $U = 1.05$ kV, the steady-state solution with a positive column and cathode and anode falls is realized and not (2). Thus, the current–voltage characteristics shown in Fig. 1b and c are for an initial charge density $q_- = q_+ = \sqrt{Q/\beta}$.

A steady-state solution of (1) exists for any voltages and any external ionization source strength. But for larger current densities a steady discharge cannot be realized because of a different kind of instability [2, 3]. However, it is known [4] that for voltages even twice as large as the breakdown voltage there is a phase of volume arcing of the discharge which is described by (1). For $d = 0.02$ cm and such voltages ($U \sim 2.7$ kV) nanoseconds are required to reach a quasistationary regime. Since the growth time of instabilities is longer, for a certain time a regime of volume discharge with characteristics shown in Fig. 1 is realized.

An estimate of the arcing time of a volume discharge is beyong the scope of the present work.

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DIRECT ELECTRIC CURRENT IN A MEDIUM WITH A LARGE NUMBER OF CRACKS

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The solution of the problem of an electrostatic field and a field of direct electric current in a medium containing a large number of randomly arranged cracks is the theoretical basis for some important methods of nondestructive checking of damage to materials (e.g., metals, rocks) [1, 2].

In the present study we propose an approach for finding the multipoint moments of a statistical solution of this problem. For the case of a direct electric current we consider a method of constructing the means (mathematical expectations) and correlation functions of the current and voltage vector fields with an electric field in a medium with cracks.

In recent years there have been a great many studies devoted to the description of the effective thermal, electrical, and magnetic properties of inhomogeneous materials (see, e.g., [3]). The problem of constructing the effective parameters of an inhomogeneous medium reduces to calculating the mean value of the random field of the solution. For some stochastic structures this problem admits of fairly good approximate, or even exact, solutions (exact summation of series of perturbation theory). However, in calculating the variance of the solution, when we construct the correlation functions in terms of which we can express, e.g., the mean energy density of the field, visible results can be obtained only for the case of weak inhomogeneity, where we confine ourselves to the first terms of the series of perturbation theory [4].

In the specific case of a medium containing a field of isolated inhomogeneities, a number of authors have used the effective (self-consistent) field method, which is well known in many-particle theory.

It should be noted that the idea of self-consistency for describing the effective properties of an inhomogenous medium can be used in various forms. In [5, 6] self-consistent solutions of an electrical conduction problem were constructed on the basis of the assumption that each typical inhomogeneity – for example, a polycrystal grain – behaves as if it were isolated in a homogeneous medium whose properties coincide with the effective properties of the entire medium, while the field in which such an inhomogeneity is situated was taken to be equal to the external field. Such a modification of the method is sometimes called the effective-medium method [7].

In the present study, in constructing a self-consistent solution for a medium containing plane elliptical cracks, it is assumed that each crack behaves as if it were isolated in a principal medium with known properties, and the presence of the surrounding cracks is taken into account by means of the effective field in which it is situated. Unlike the usual formulations of the method, in which the effective field is chosen to be the same for all particles [8], here we assume that this field is random, varying from crack to crack. To construct the equations which will be satisfied by the statistical moments of the effective field, we make use of a procedure of the "smoothing" type [9], in which the chain of equations connecting all the multipoint moments of the solu-

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